Graph Theory Applications in Video Games

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COSC 594 – 2020/03/11

Questions

- Given a 3D model M of n vertices, how many triangles are drawn if done via Triangle List?
- What is the $lg^*(2^{2^{65536}})$? Alternatively, what is the $lg^*({}^{6}2)$?
- What does bitDP stand for?

About Me

About me

- Master's Student on Course-Only track.
- Did undergrad at UTK. Graduated in Spring 2018.
- Hobbies
  - Game/Web Development
  - Content Creation (Music & Video)
- Born in Knoxville, TN! Look outside a window for a picture if you want.

More on me!

- Been Programming since I was 6. I like to do side projects on the side.
- Not actually a gamer.
- Been a TA here for around 4 years.
- Outside of Computer Science, my goal is to become a polyglot of Asian Languages.
- Not a pet person… (But I prefer cats btw)

Showcase - Game Development History

- Involved since mid-2008
- Worked with other Indie teams
- In-house Engine Development.
- 2014 – Solo Project: “Keyboard Hero”
  - Rhythm Game like Guitar Hero
  - Released on Gamejolt
  - Coded in GML, Delphi, and C++
  - Over 63,000 views and 16,000 plays

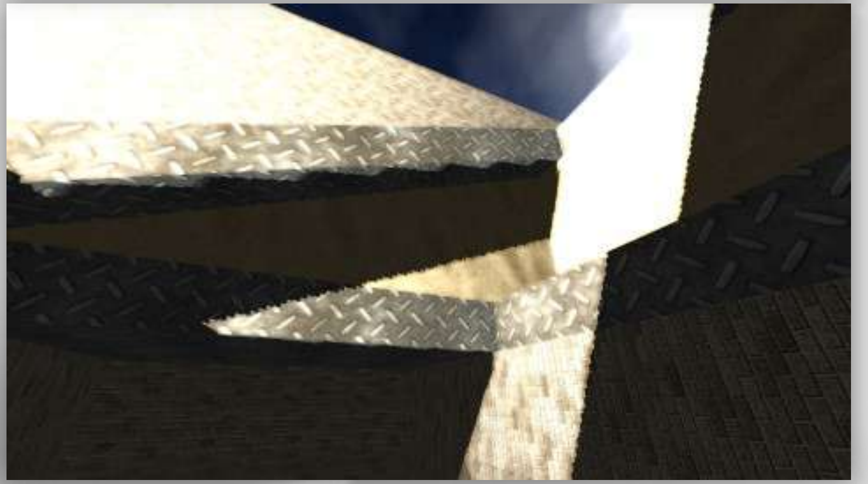
Showcase - Keyboard Hero V7.5

Showcase - Game Development History

- 2017 – Solo Project: “Project RX”
    - Successor to previous game.
    - Had composers create music specifically for the game.
    - Over 20 songs charted.
    - Engine written in C++ entirely from scratch.
    - Unreleased as of 2020.

Showcase - Game Development History

- 2017 – CN_GL (Clara Nguyễn's WebGL Wrapper)
  - Concept 3D engine written entirely from scratch to be playable in your web browser.
  - Written entirely from scratch in 52 hours.
  - This is playable! http://web.eecs.utk.edu/~ssmit285/vORIcEmA/finalp/



Why game dev experience matters

- It’s one thing to play games. It’s another to develop them.
- Code can’t be written sloppily. Usually has to generate and draw 30 -60 frames onto your monitor on modern hardware.
  - It’s *extremely* obvious when a game is poorly optimised.
- There’s lots of unique problem solving in Game Dev. You often build a “toolbox” of ways to approach a problem over time.
- Relevance-wise, Graph Theory plays a huge role in game development.

Disclaimers

- This is not your average talk.
- This is a Graph “Theory” talk… I only give a handful of game mentions and stick to concepts.
- Algorithm discussion is minimal. If I mention an algorithm, then I’ll tell you what it should do, not go over the procedure (except for DFS).
- Topics are laid out intentionally to where they all may not be discussed.
    - All topics and details are here: https://tiny.utk.edu/talk5

Outline

- *The Warmup* – "Rules" of Graphics
- Racing Games – Lap Counting
- Maze Generation – Disjoint Sets & Union-Find
- Hamiltonian Path Detection – bitDP
- Honourable Mentions
- Discussion

“Rules” of Graphics

The “rules” of graphs of computer graphics

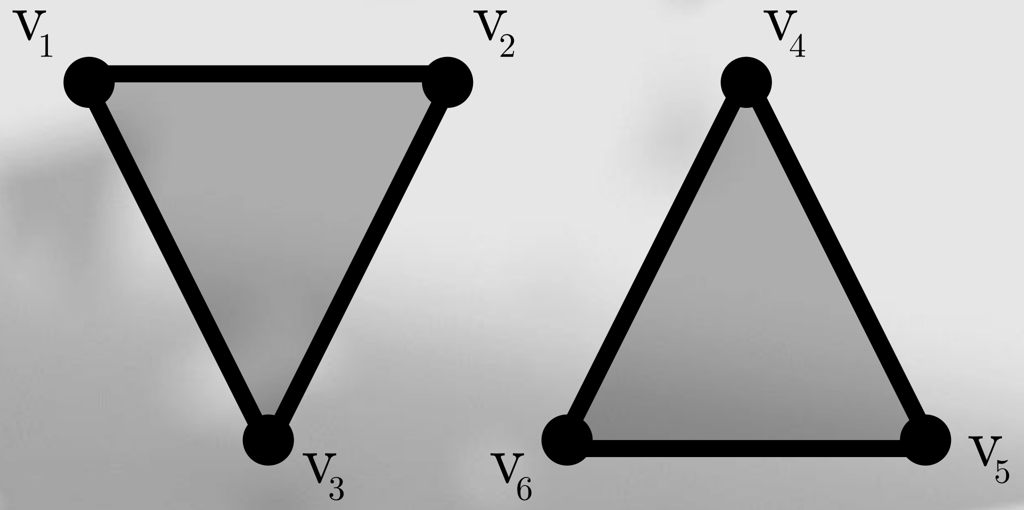
- Unlike most graphs we dealt with in class, the rules change here:
  - Vertices have **positional** coordinates (x, y, z) to define position in space.
  - There is only **one** way to represent graphs in space.
  - Edges (connections between vertices) are **implied**.
  - Everything is oriented around **triangles**.

The “rules”: Edge Implication

- Edge Implying depends on how we tell the computer to draw.
- Several modes. Here are the common ones:
  - **Triangle List:** Every 3 vertices form a triangle.
  - **Triangle Strip:** First 3 vertices form a triangle. Every new vertex after will form a triangle with the previous 2 vertices.
  - **Triangle Fan:** First vertex is in every triangle. Each set of 2 vertices after the first form a triangle with the first vertex.

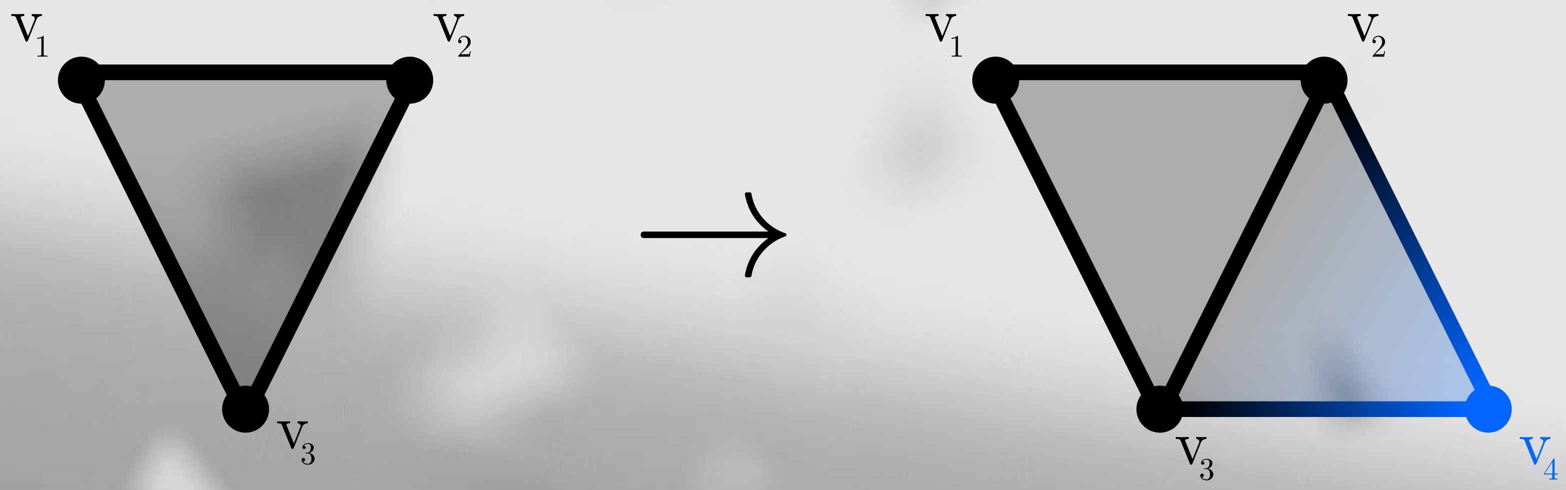
The “rules”: Triangle List

- Naïve triangle drawing in multiples of 3.
- $n/3$ triangles drawn.
- Assume we are given a model m where $V(m) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$



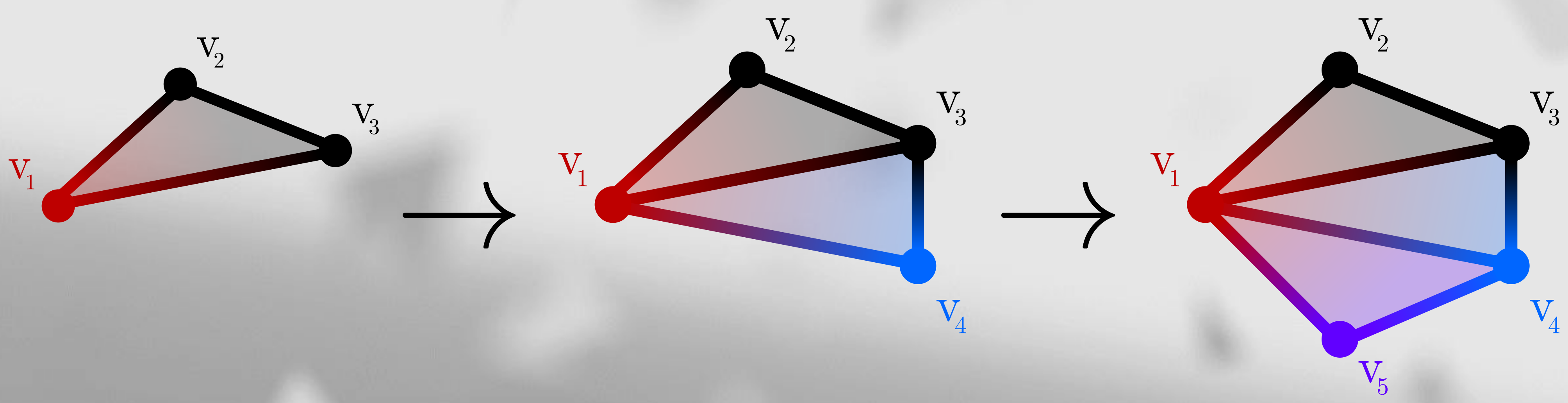
The “rules”: Triangle Strip

- Uses previous 2 vertices & new one to form triangles.
- $n \geq 3$. $n - 2$ triangles drawn.
- Assume we are given a model m where $V(m) = \{v_1, v_2, v_3, v_4\}$



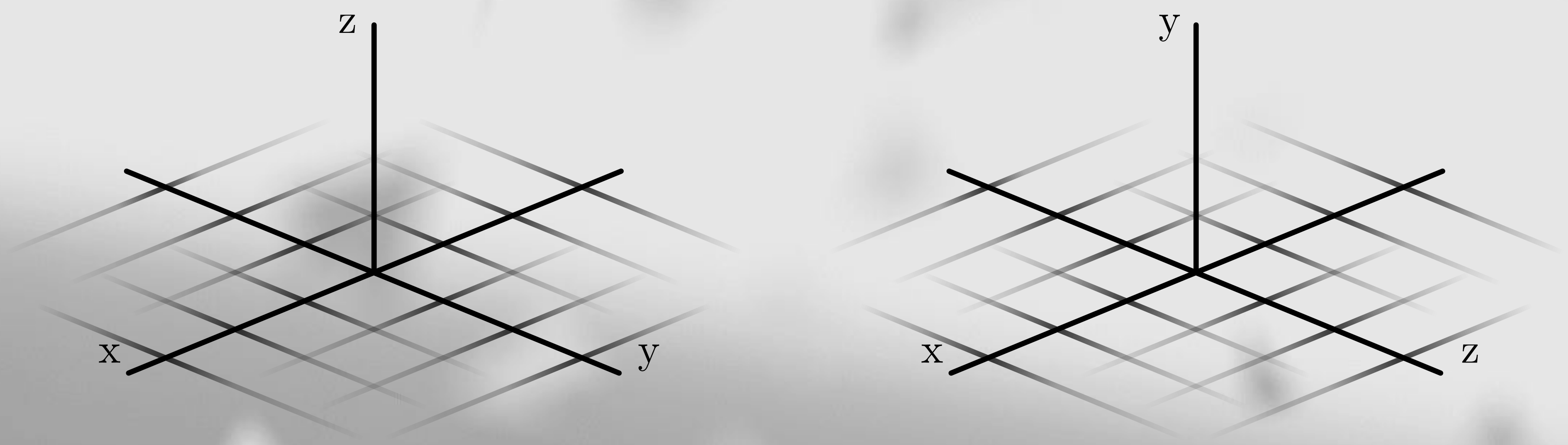
The "rules": Triangle Fan

- Uses first vertex and latest 2 vertices to form triangles.
- $n \geq 3$. $n - 2$ triangles drawn.
- Assume we are given a model m where $V(m) = \{v_1, v_2, v_3, v_4, v_5\}$



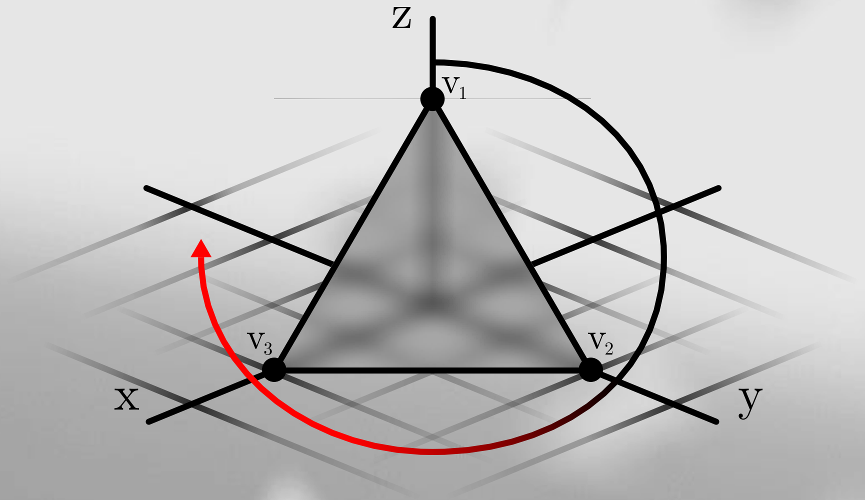
The “rules”: Coordinate System

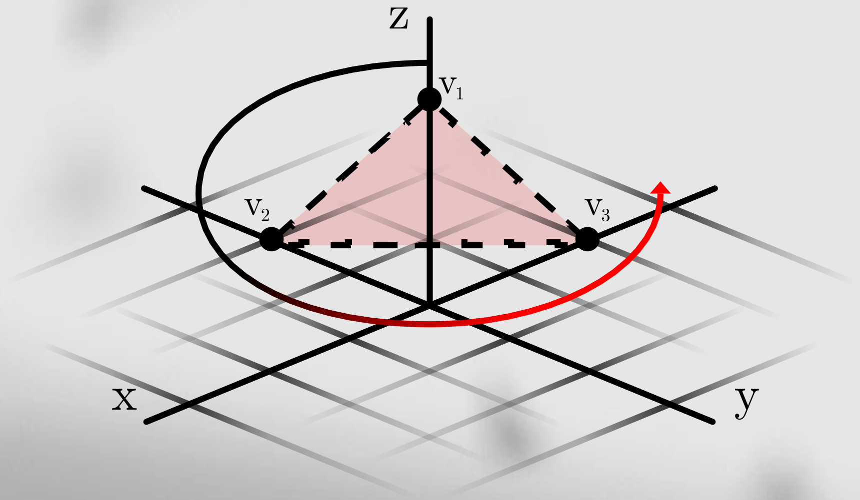
- Two of the most popular cartesian coordinate systems for 3D space:
    - (x, y, z) where $\boldsymbol{z}$ is the height axis
    - (x, y, z) where $\boldsymbol{y}$ is the height axis



The “rules”: Back-Face Culling

- Front side has a triangle. Back side is invisible due to **back-face culling**.
- Relies on the order we draw the vertices. Vertices with order being **clockwise** is front-facing. **Counter-clockwise** is the back.





Lap Counting

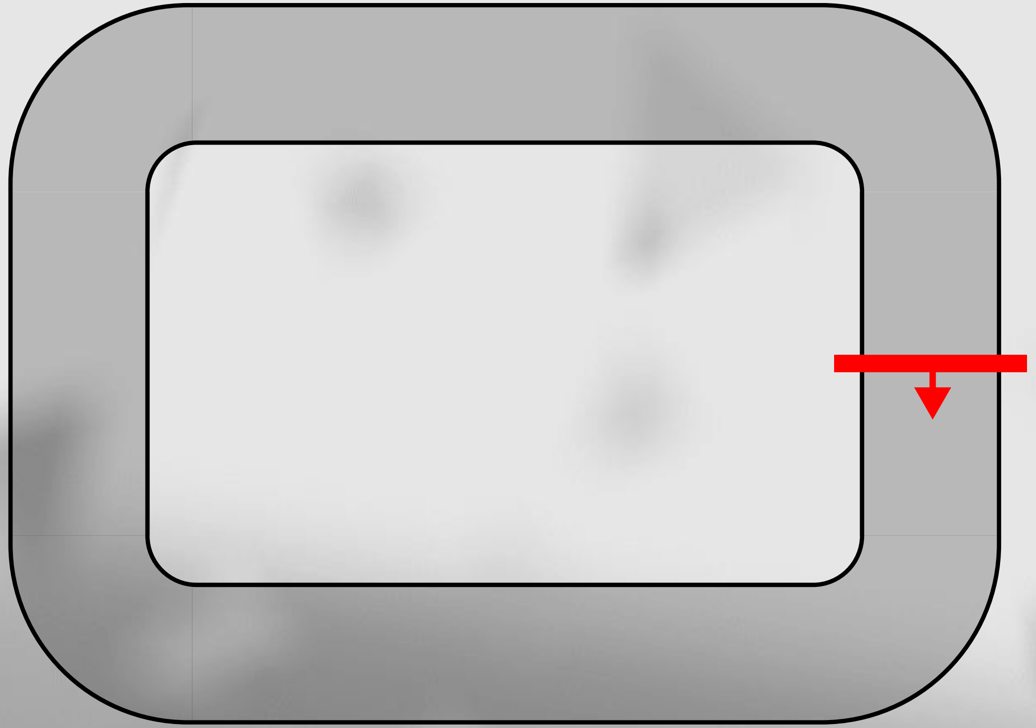
Racing Games

Lap Counting – The Basics

- A racing game must keep track of a few things…
  - Player Lap
  - Player Position
  - Distance between players
- How do games know when a player has completed a lap?

Lap Counting – The Basics

- Assume the following (extremely simple) racetrack:

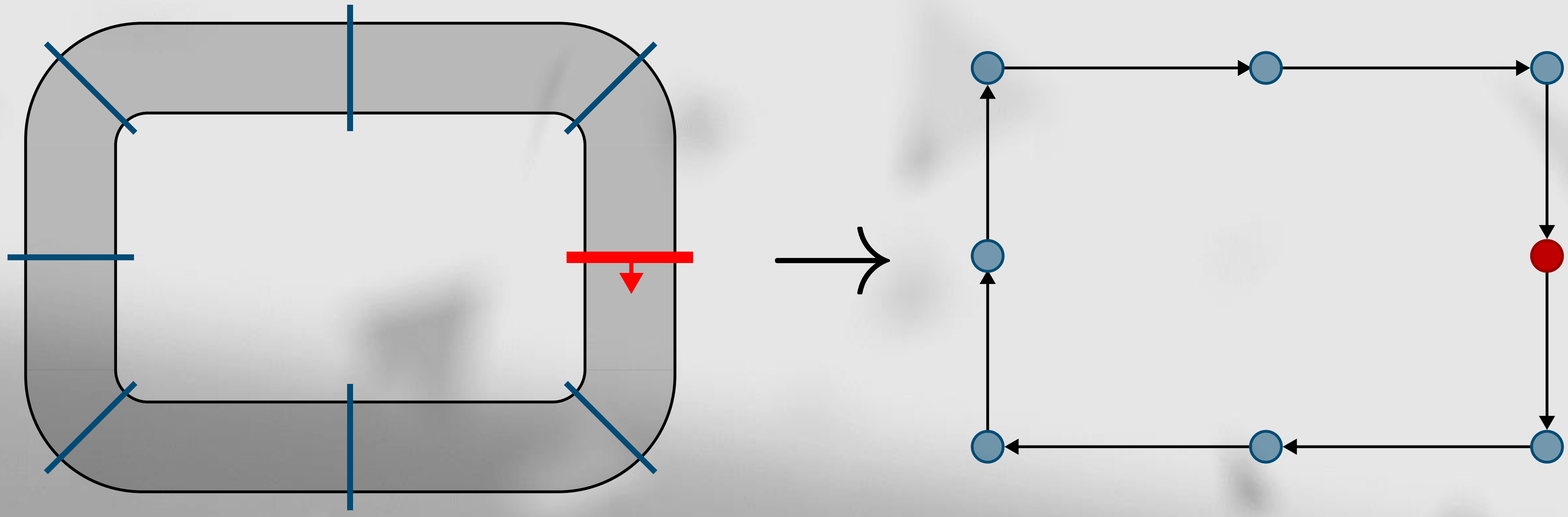


Lap Counting – The Basics

- Divide the track into “checkpoints”.
- Players will have to hit all “checkpoints” and the finish line for a lap to count.
- This can be implemented as a directed graph where all checkpoints are vertices and a complete lap is a **Hamiltonian Circuit**.

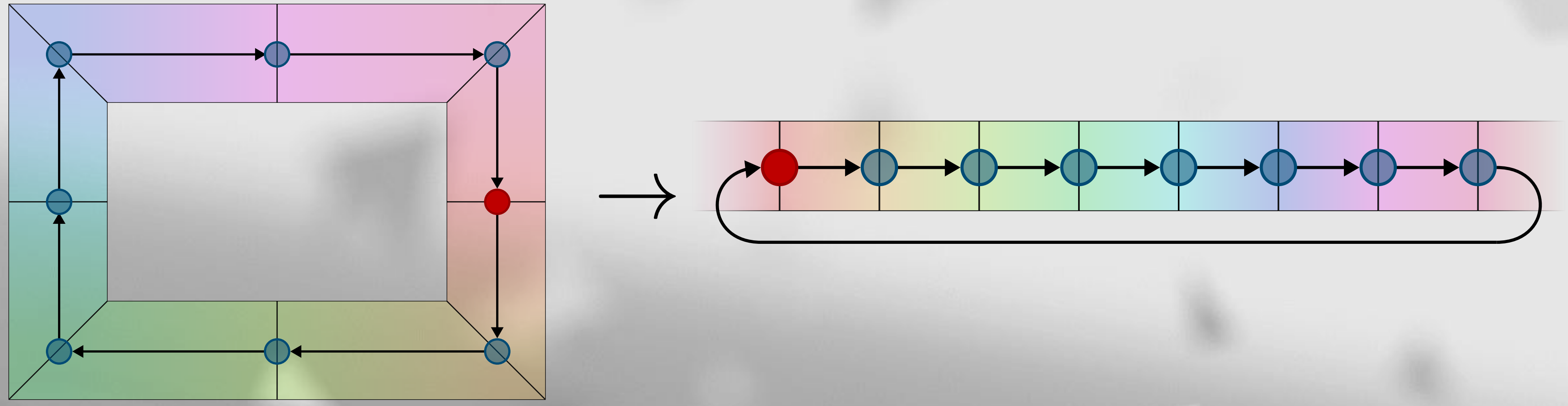
Lap Counting – The Basics

- Simple racetrack broken up into checkpoints, and as a directed graph:



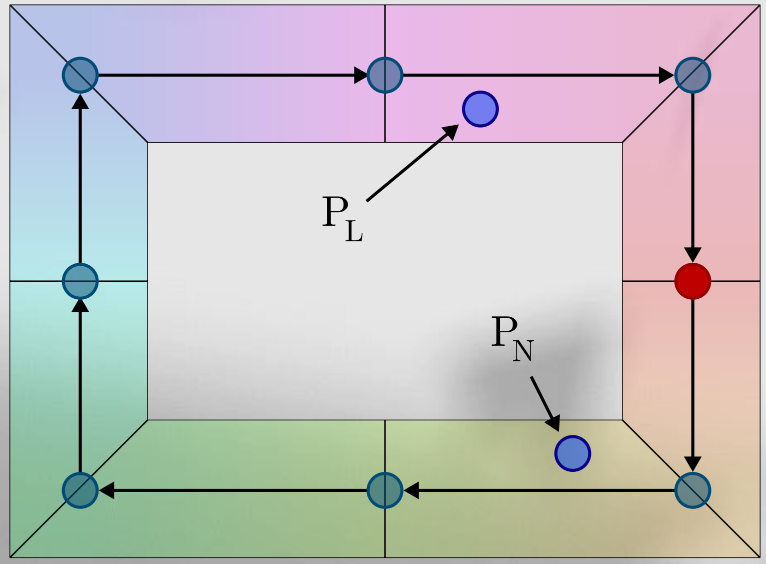
Distance between players

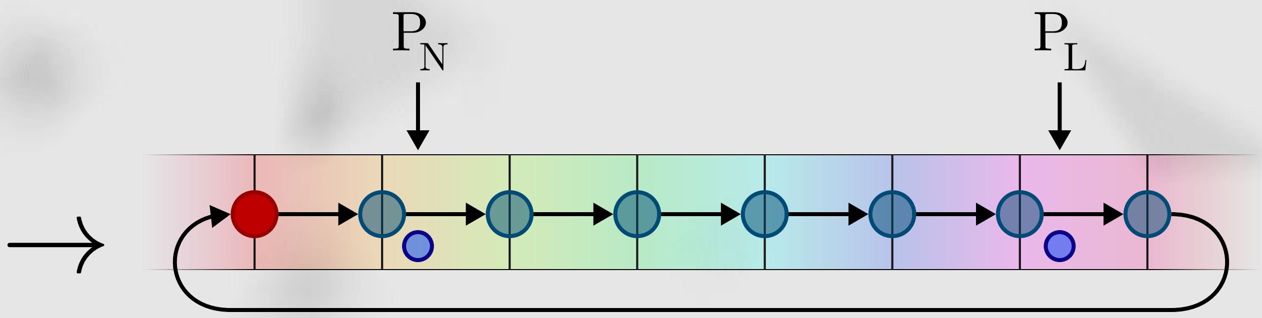
- How will we know how far someone is from first place?
- Graph is broken segments by-vertex, rearranged into a straight line with circular ending node, making distance computation extremely trivial.



Distance between players

- So let's say Dr. Langston (P_L) having a really good race... unlike me (P_N)...

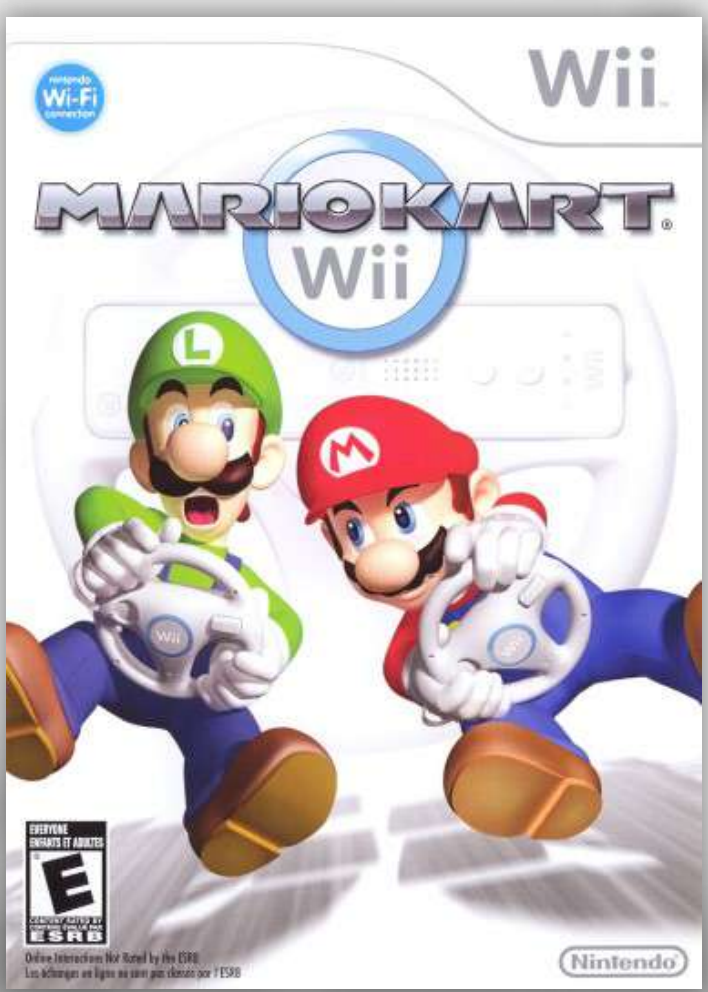




Lap Counting – Breaking the Rules

- In practice, there are other ways to do lap counting… besides Hamiltonian circuit detection.
- They flopped. Let's look at an extreme example.

Mario Kart Wii for Nintendo Wii (2008)

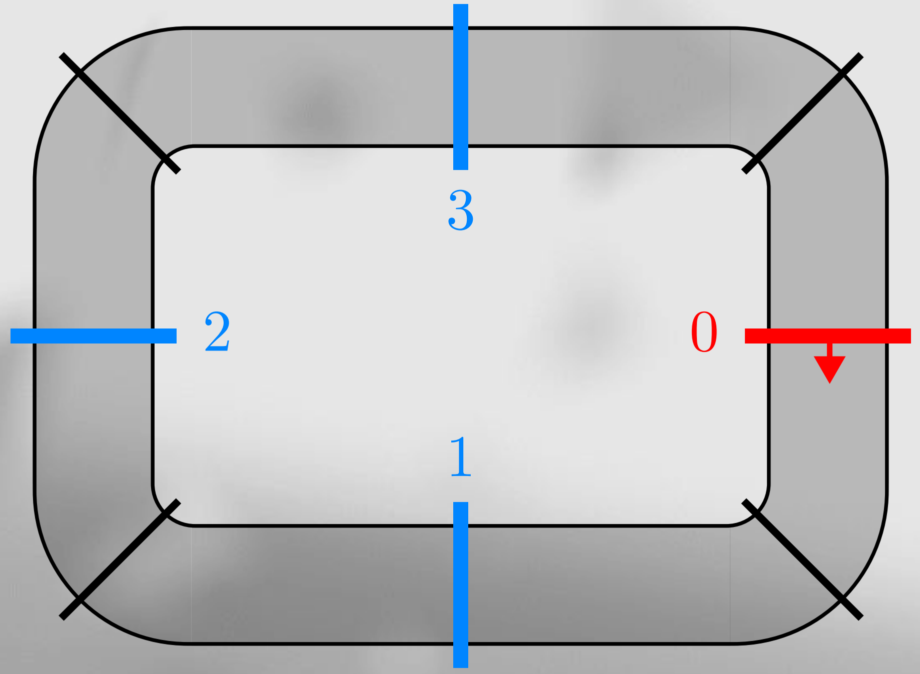


Mario Kart Wii – Breaking it down

- Breaks track into **spawn checkpoints**.
  - If you fall out of the track, you spawn at these.
- Breaks track into **key checkpoints**.
  - Finish line also counts as a key checkpoint.
  - Tells where you are and if you completed the lap… or do they?

Mario Kart Wii – Breaking it down

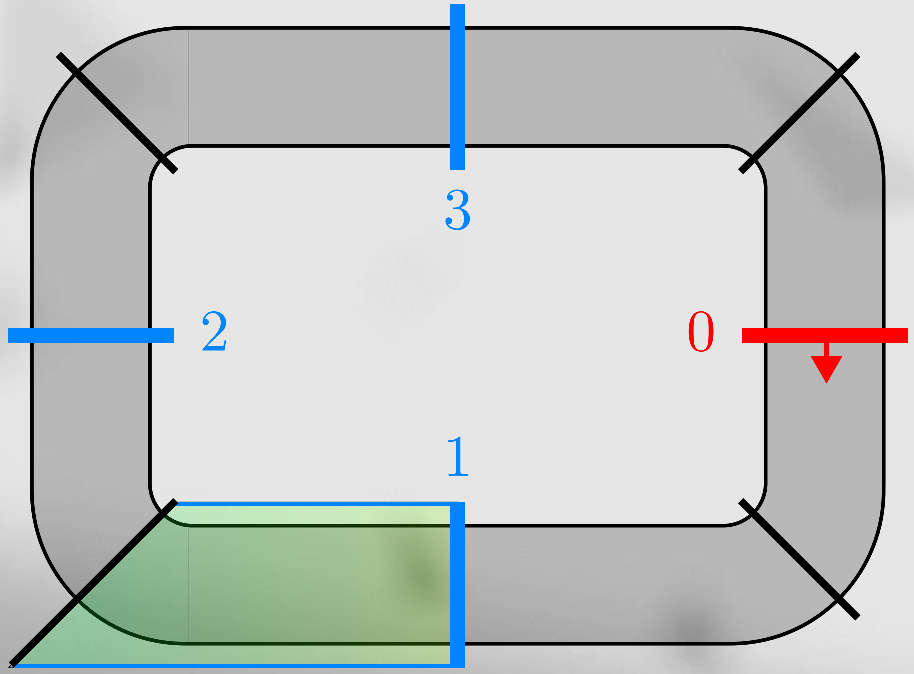
- Assume the following (extremely simple) racetrack with **key checkpoints**, **spawn checkpoints**, and a **finish line**:



Mario Kart Wii – Breaking it down

- Going between a **key checkpoint** and a next checkpoint (**spawn**, **key**, **finish**) updates where you are in the track.

- **Example:** Hitting between 1 and the spawn checkpoint right after will register as you passing through checkpoint 1.

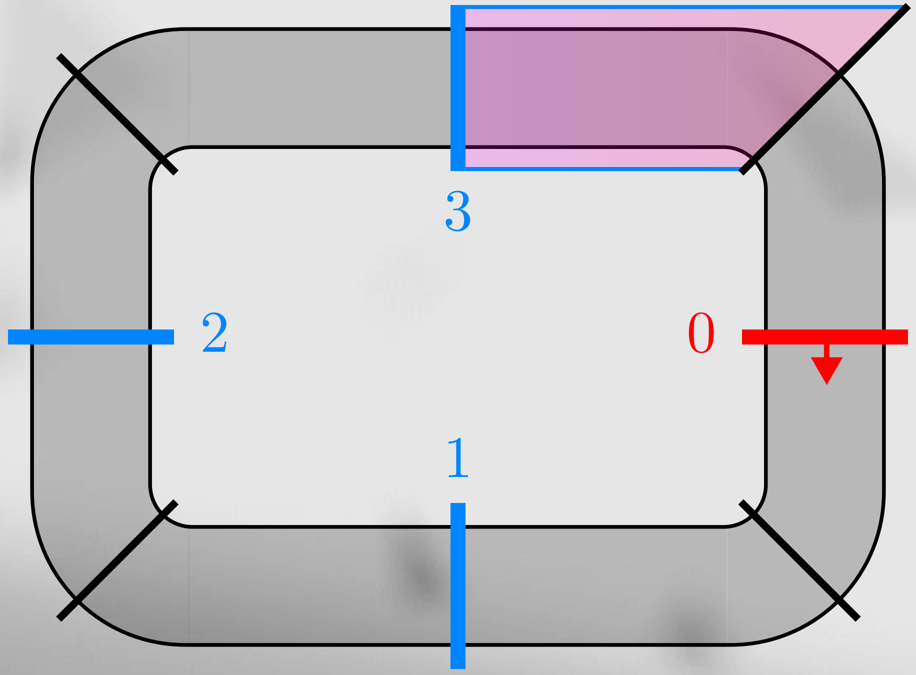


Mario Kart Wii – Ultra-Shortcuts

- **Critical Flaw:** Game allows you to hit the next, current, *and previous* **key checkpoints**. Completing a lap requires hitting **only the last one**.

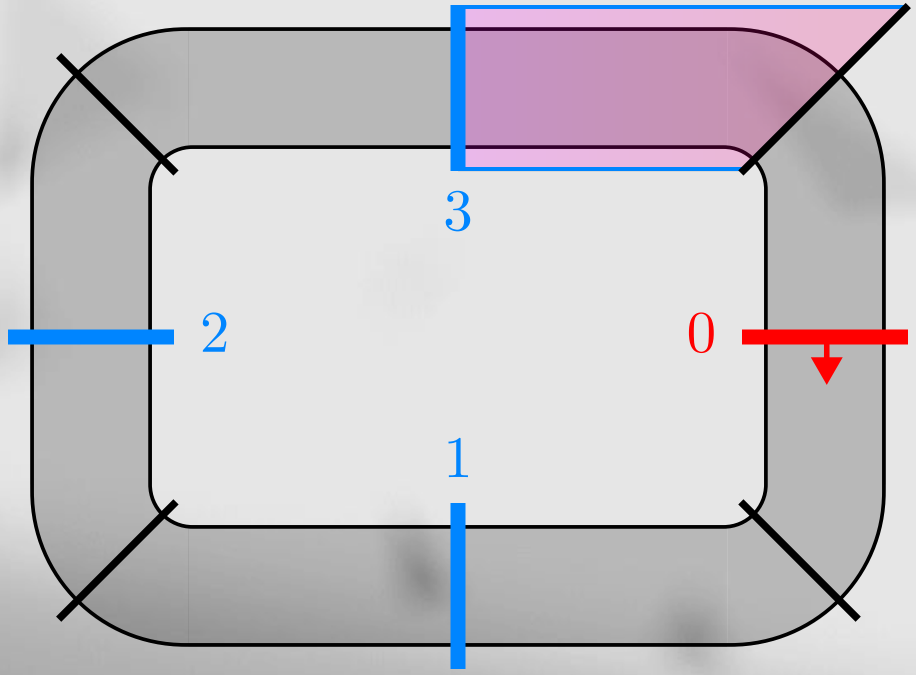
- From the start of the race, we can avoid going through **1** and **2**. Just jump to **3** and drive up to **0**. The lap will count.

- This is known as an **Ultra-Shortcut**.



Mario Kart Wii – Ultra-Shortcuts

- These are not as simple as driving backwards though.
- Going in reverse from the **finish line** will deduct 1 from your lap count. Detected by the **spawn checkpoint** right behind.
- Usually involves finding glitches or out-of-bounds areas to jump to **3**.



Mario Kart Wii – Ultra-Shortcuts

- The *normal* world records didn't last very long after that…

2008-06-01	1′35″799	Ridley
2008-06-01	1′29″550	Ridley
2008-06-01	1′26″078	Alvin
2008-06-01	1′03″520	Ridley
2008-06-01	0′43″912	Alvin
2008-06-01	0′42″446	Ostro

→

2019-09-13	0′17″100	Ejay
2019-09-19	0′16″852	Ejay
2019-09-23	0′16″691	Niyake
2019-09-23	0′16″591	Niyake
2019-09-23	0′16″385	Niyake
2020-01-12	0′16″332	Niyake

- **Moral of the Story:** Use Hamiltonian Circuit detection for lap counting.

Maze Generation

Disjoint Sets & Union-Find

An observation of mazes

- Cells matched with a select few of adjacent cells.
- Others are separated by “walls”.
- Can be represented as a graph. Depending on properties of the maze, it can be a minimum spanning tree.
- We can use DFS (Depth-First Search) and BFS (Breadth-First Search) to traverse the maze to find a solution easily from any S to any T.

Disjoint-Sets

- Sets that have no element in common.
- “Mazes” with every wall put up is a good example, as no cell is connected.
  - Basically a graph without any edges connecting any nodes.
- We have operations: **union** and **find**:
  - **Union:** Join two disjoint sets together.
  - **Find:** Get the ID of the set that a cell belongs to.

Disjoint-Sets – Continued

- **Union:** Join two disjoint sets together.
  - Notated as $union(i, j)$ where $S_i = S_i \bigcup S_j$.
  - In English: All vertices in S_j move into S_i. Then, S_j is *deleted*.
- **Find:** Get the ID of the set that a cell belongs to.
  - Notated as $find(i)$ where i is a cell ID.
  - More on this in a bit…

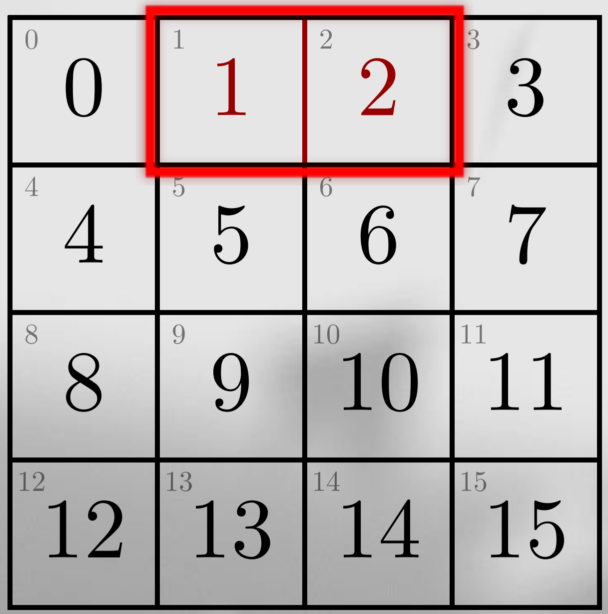
Disjoint-Sets – Example

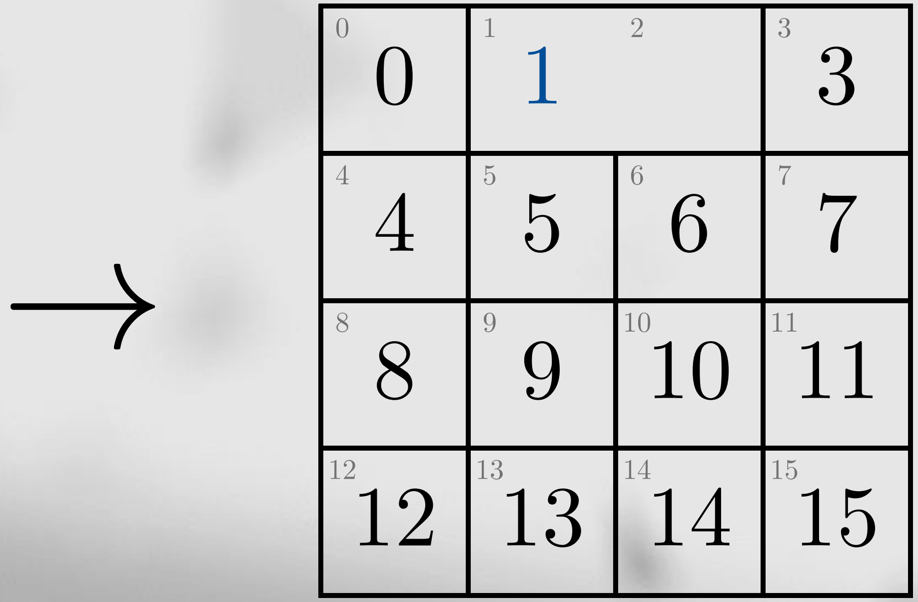
- Assume a graph M where $n = 16$, and $m = 0$. Each separate vertex is part of its own set $S_i(v_0 \in S_0, v_1 \in S_1, ..., v_{n-1} \in S_{n-1})$. Show as a 4×4 grid:

0 0	1 1	2 2	3 3
4 4	5 5	6 6	7 7
8 8	9 9	10 10	11 11
12 12	13 13	14 14	15 15

Disjoint-Sets – Example

- Let's do $union(1, 2)$. Notice how the walls break down between the two. They have an edge between them. Now $S_1 = \{1, 2\}$ and S_2 is deleted.

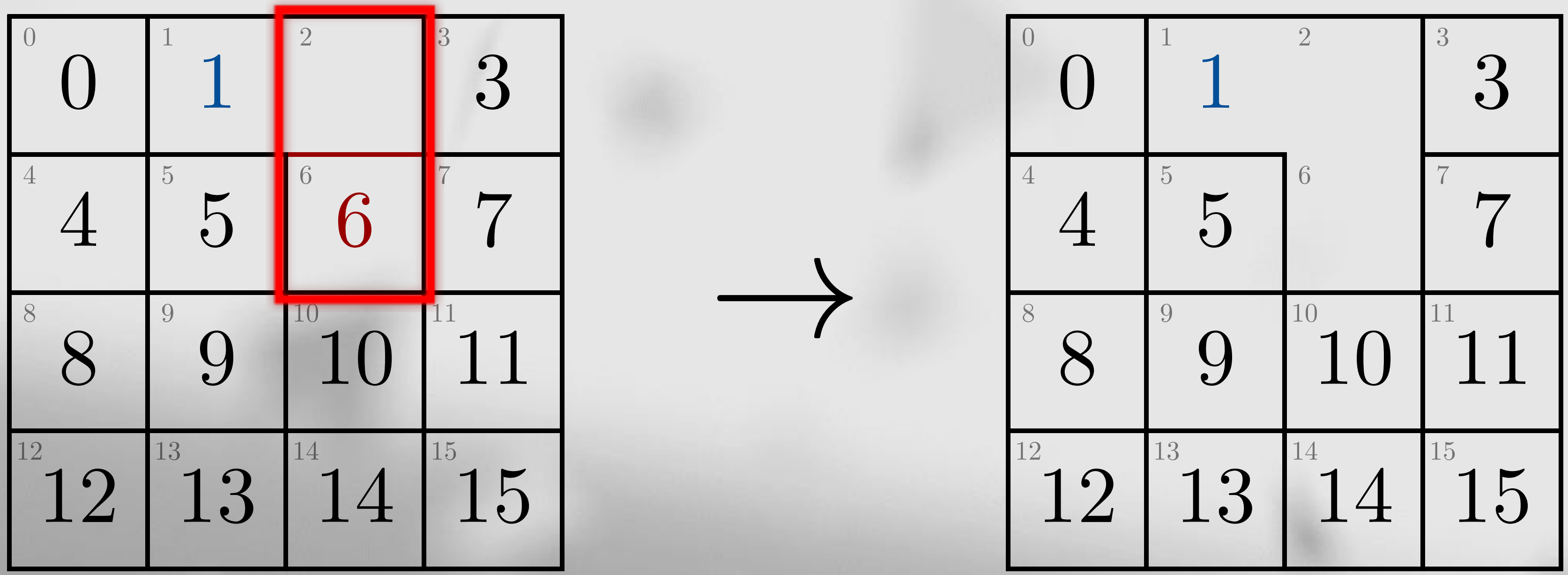




Disjoint-Sets – Example

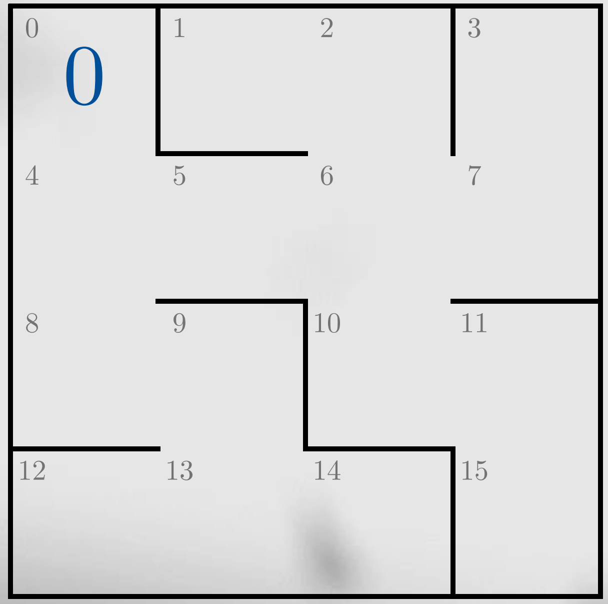
- Let's do $union(2, 6)$. Break down the wall between where 2 used to be and 6.

  Now $S_1 = \{1, 2, 6\}$ and S_6 is deleted.



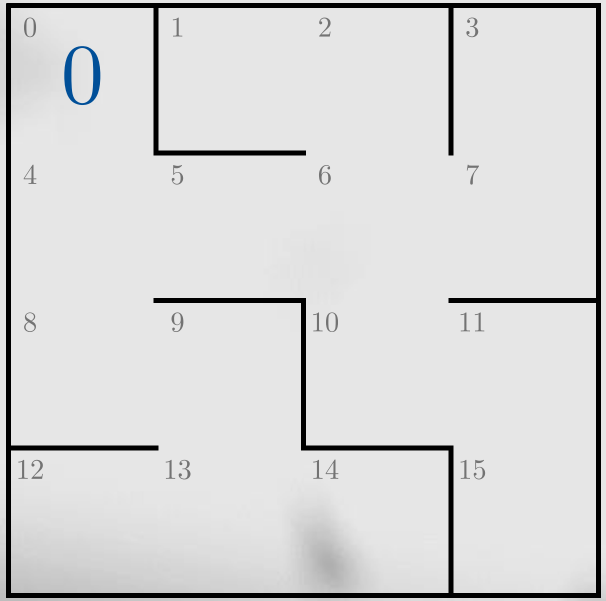
Disjoint-Sets – Example

- To properly generate a maze:
  - Repeat the procedure on cells that are adjacent but are in different groups.
  - Do this until there is only one group left... $S_0 = \{v_0, v_1, ..., v_{n-1}\}$



Disjoint-Sets – Some properties

- Known as **Randomised Kruskal's algorithm**.
- There are no cycles.
- There is one path from every S to every T.
- Tends to generate mazes with patterns that are easy to solve.
- If shown as a graph, it's a minimal spanning tree.

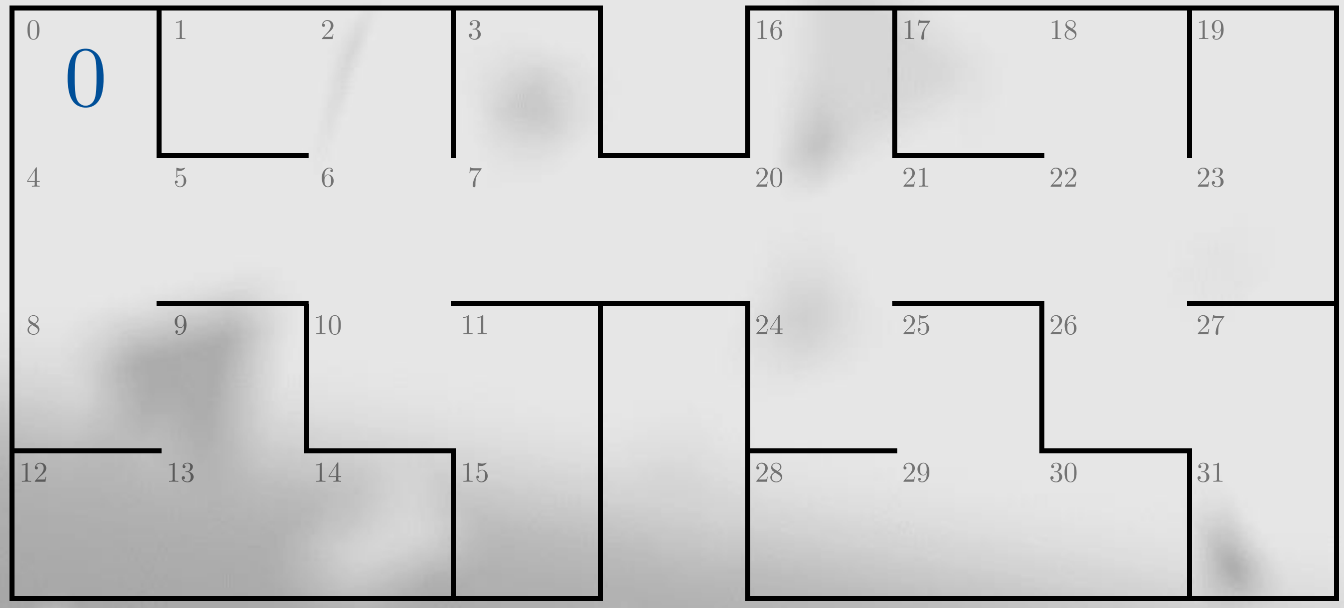


Disjoint-Sets – We can do better

- A simple maze is boring.
- We can connect 2 together by breaking down a wall between them (or even adding a “hall” between them).
- Any cell in one maze is always accessible from any other cell. Connecting like this keeps this property intact as we can always go toward the “hall”.
- This makes more complex, interesting, non-square puzzles.

Disjoint-Sets – 2D Expansion

- Horizontal Expansion. Notice how there is always a path from the left maze to the right maze since we can always access 7 and, thus, the “hall” to 20.

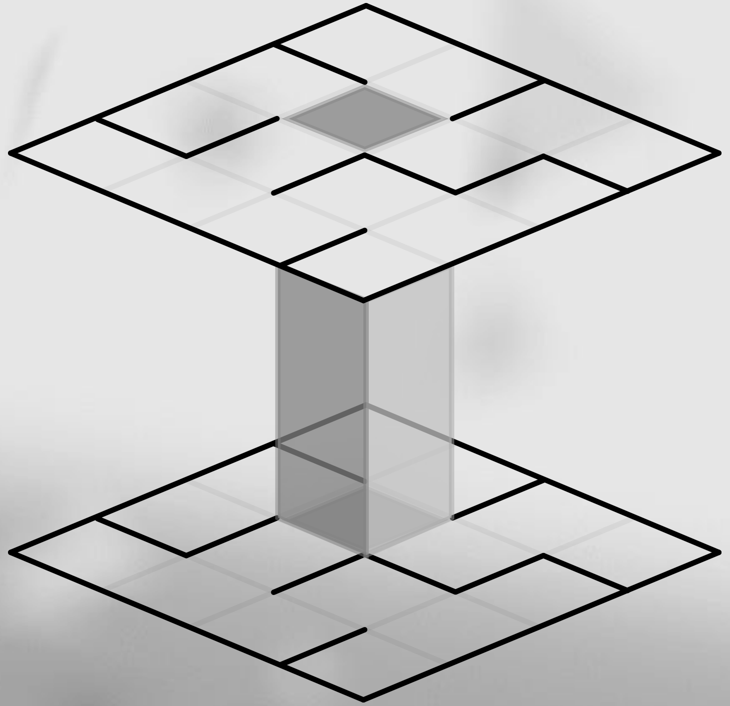


Disjoint-Sets – We can still do better

- We can expand a dimension (or a few).
- Connect 2 mazes together by making a cell have an “elevator” to go up.
- Same property from before still holds. There will always exist a path from one cell to another, even when going up to another floor.

Disjoint-Sets – 3D Expansion

- Floor Expansion. Again, notice how there is always a path from every cell to every other cell.

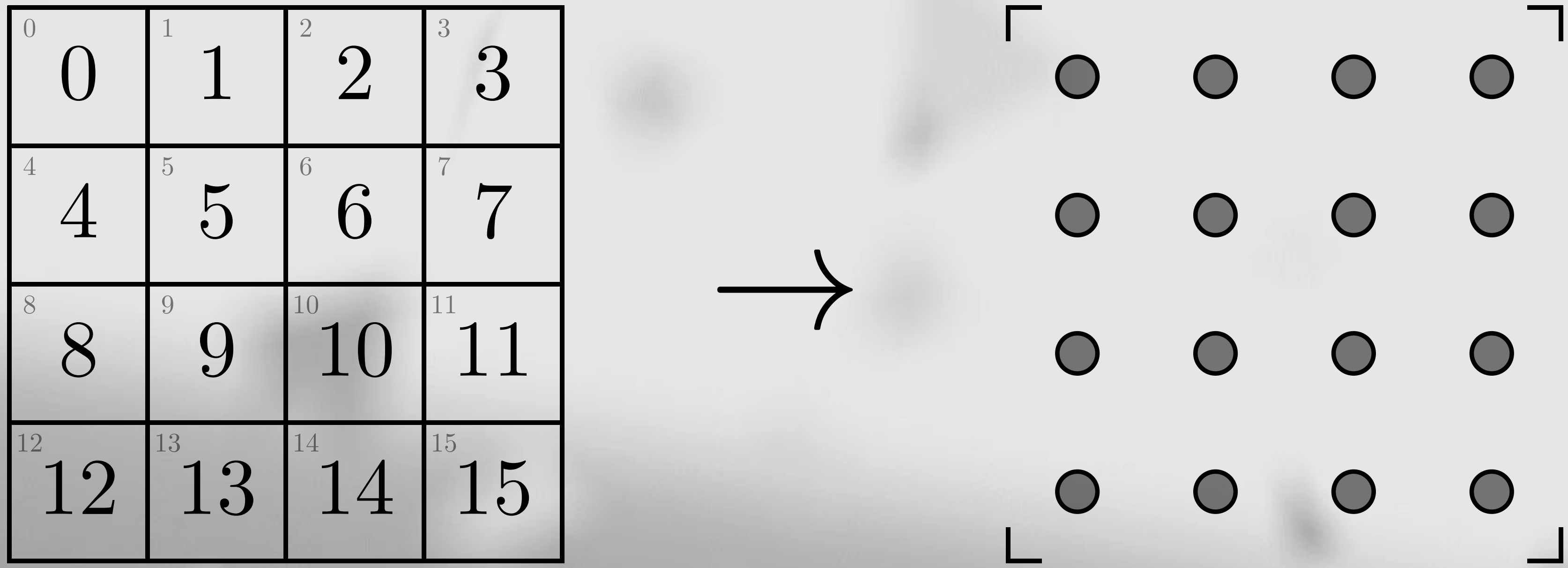


Disjoint-Sets – Find operation

- In theory, $union(i, j)$ on two sets is trivial. To a computer, it requires work.
- **Find:** Get the ID of the set that a cell belongs to.
  - Notated as $find(i)$ where i is a cell ID.
  - Interpret the set as a graph.
  - Go up to root of the “graph”. That is the set’s ID.
  - When doing a $union(i, j)$, the ID of two node’s set IDs must be different or else a cycle will occur. The lowest index (rank) becomes the new root.

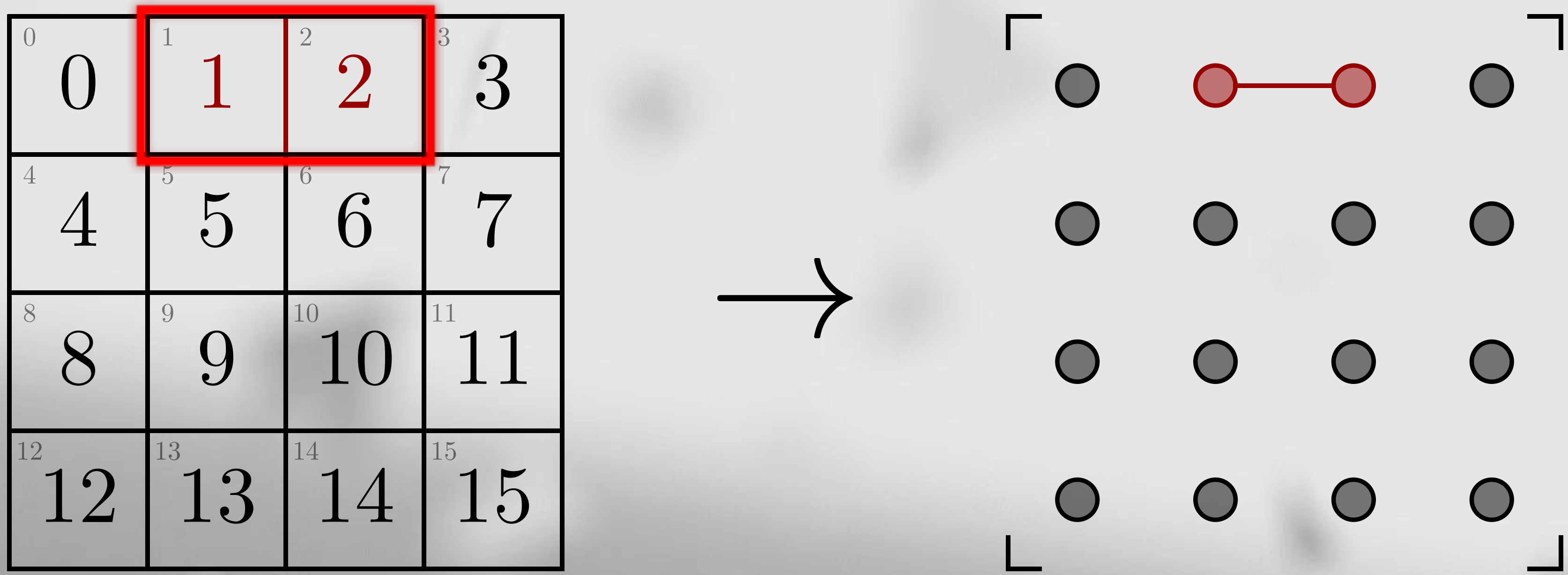
Disjoint-Sets – Find operation

- Interpret maze M as a traditional graph with vertices and edges.



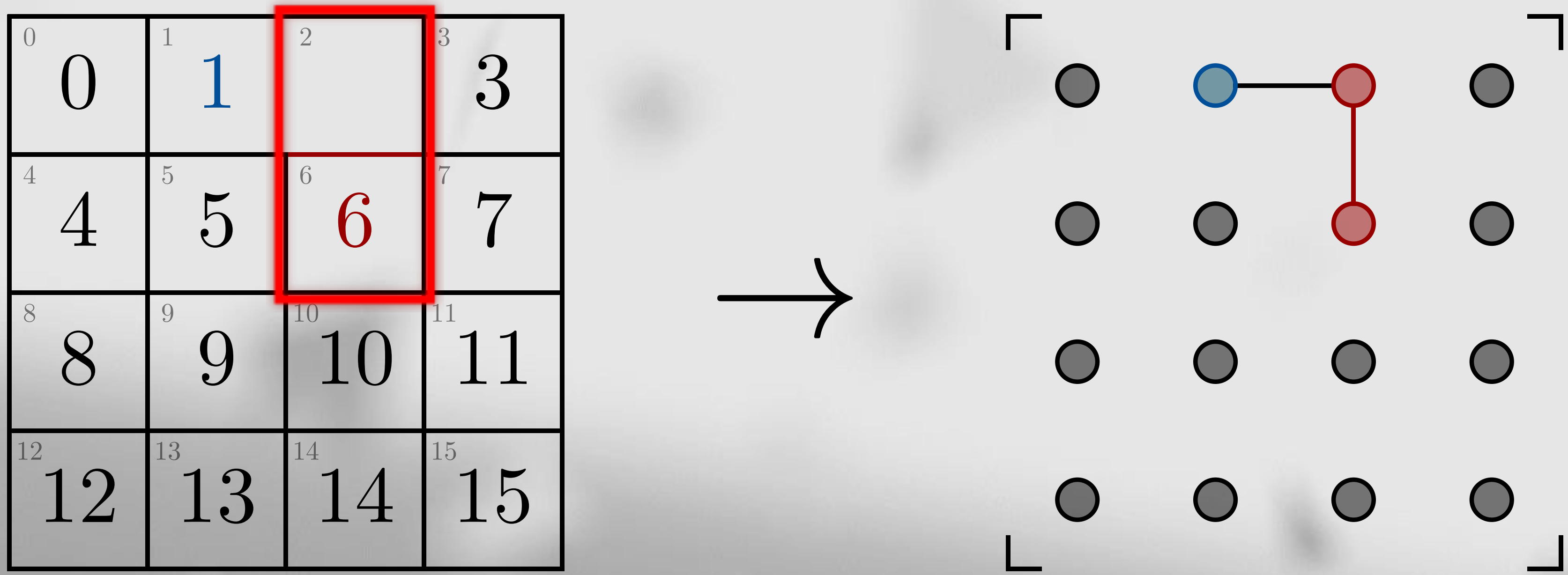
Disjoint-Sets – Find operation

- Let's do $union(1,2)$. Then, $find(2) = 1$ as $v_2 \in S_1$.



Disjoint-Sets – Find operation

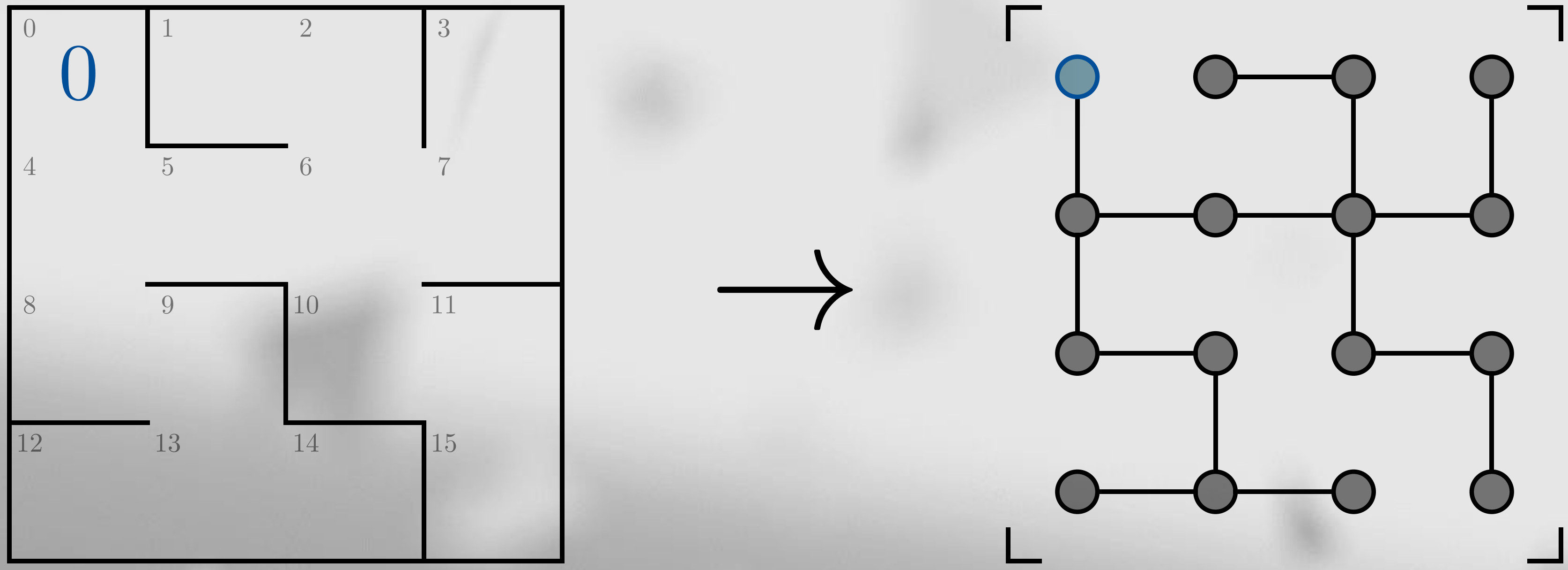
- Okay, now do $union(2, 6)$. Then, $find(6) = 1$ as $v_6 \in S_1$.



Disjoint-Sets – Find operation

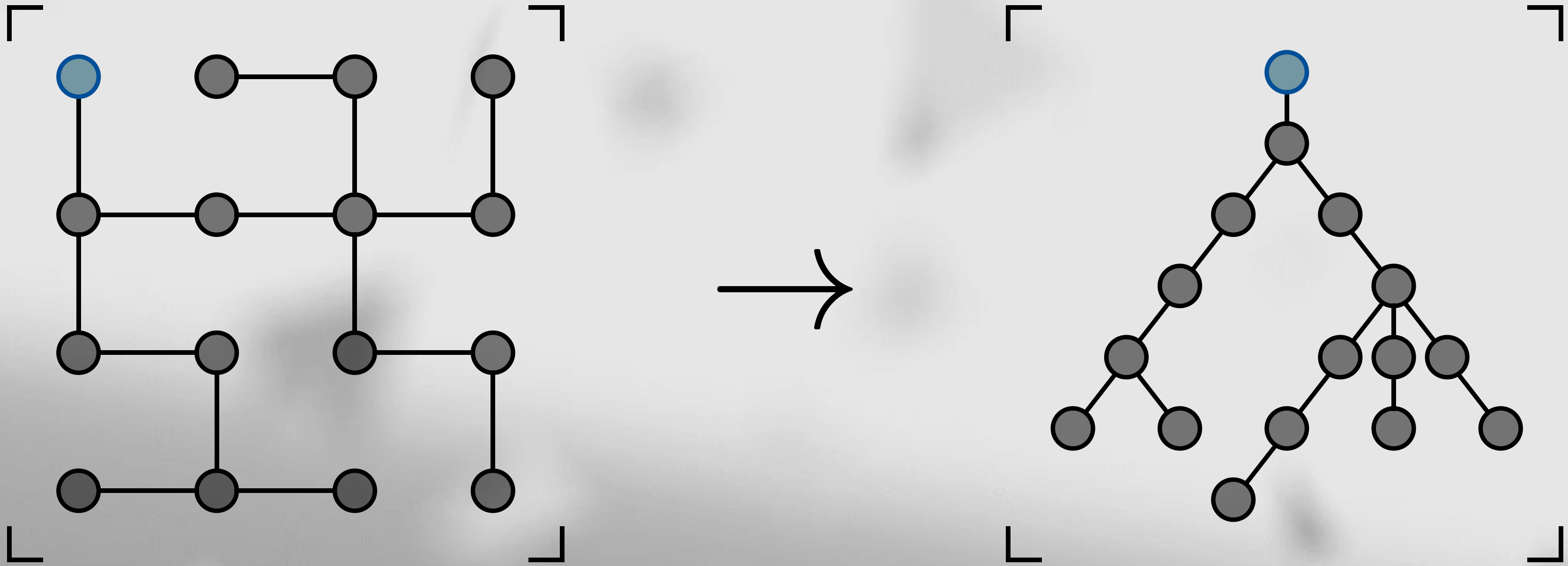
- Keep building the minimum spanning tree until entire graph is connected.

  For every vertex in the final graph, $find = 0$ as they are all in S_0 .



Disjoint-Sets – Find operation

- This can become bad quickly… The vertex at the bottom right of the maze has to traverse through $\mathbf{6}$ vertices to reach the root.



Disjoint-Sets – Find operation

- As usual, we can do better… *much better*.
- Let’s apply two concepts: **Union by rank** and **Path compression**.
  - **Union by rank** – Attach shorter tree to the root of the taller tree.
  - **Path compression** – Make every node point straight to the root.

Disjoint-Sets – Find operation

- The original lookup speed requires around n lookups to reach the root.
- With our optimisations in place, it becomes $\lg^* n$ (iterated logarithm base 2).
- In the world of Computer Science, this is essentially **constant time**.

^{n}a	x	$\lg^* x$
$^{1}2$	2	1
$^{2}2$	4	2
$^{3}2$	16	3
$^{4}2$	65536	4
$^{5}2$	2^{65536}	5
$^{6}2$	$2^{2^{65536}}$	6

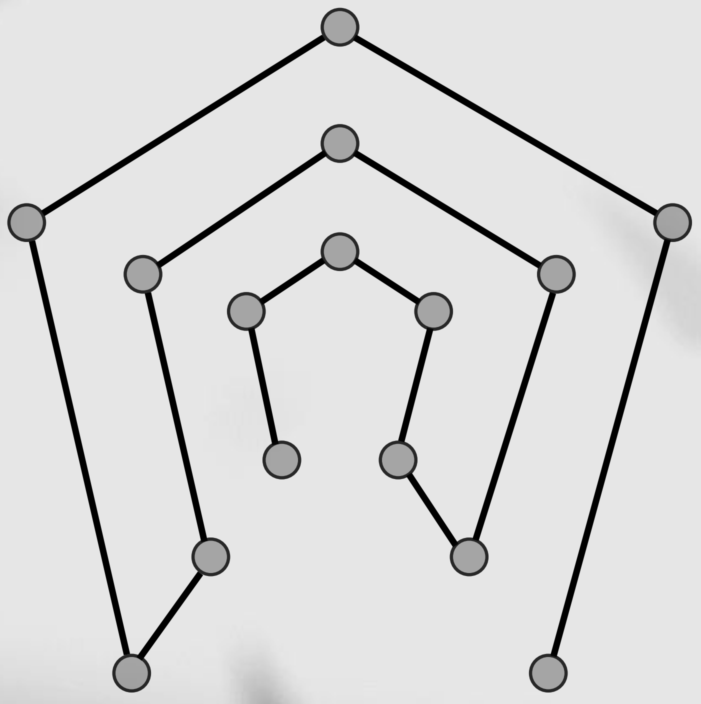
bitDP

Hamiltonian Path Detection

Some of you may have seen this before…

Hamiltonian Paths

- A path where we visit every vertex once.
- NP-Complete.
- For computers, naïvely finding these in a graph of size N **explodes** into $N!$ steps.
- Detection useful for a game generating random paths and needs to check for correctness before giving to the player.



Naïve Brute-Force Method

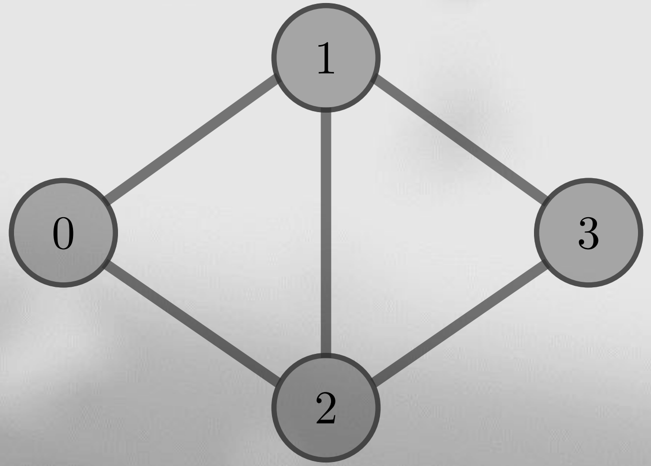
- Perform a DFS (Depth-First Search) from the starting vertex S search around all possible combinations of paths until we find a Hamiltonian Path.
- Gets the job done, but is nowhere near efficient.

DFS Breakdown

- Assuming a graph G, keep a list $V'(G) = \{\}$ which is the path (in the order we visited the vertices). Mark all vertices as **unvisited**.
- Behold the procedure $DFS(v)$. Run it on $DFS(S)$:
  1. Mark v as **visited** and add it to the end of $V'(G)$.
  2. Go through every **unvisited** vertex v' that v is connected to and do $DFS(v')$.
  3. If the size of $V'(G)$ is equal to the number of vertices in G, a Hamiltonian Path exists!
  4. If one wasn't found, remove v from $V'(G)$, mark it as **unvisited**, and go back to the previous call of the procedure.

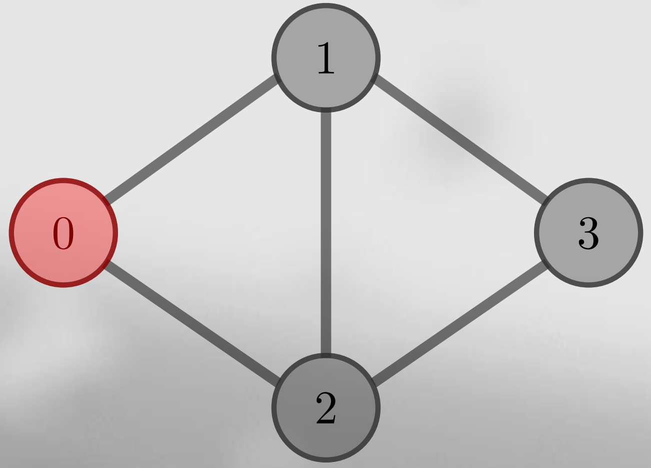
DFS Example - Setup

- Behold a graph G where $S = v_0$ and $V'(G) = \{\}$. Find if a Hamiltonian Path exists starting from S via $DFS(S)$.



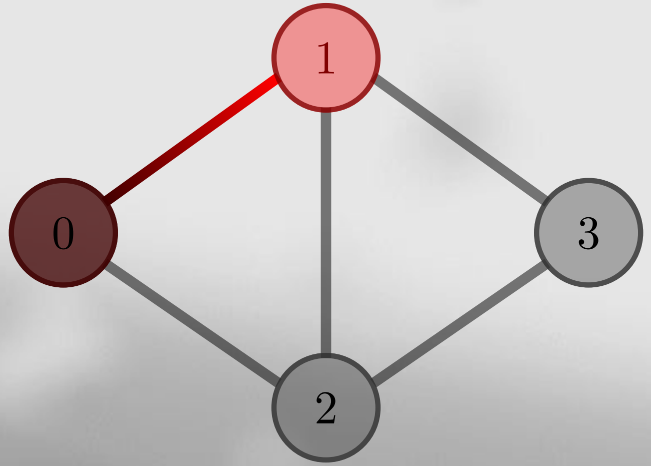
DFS Example - $DFS(S)$

- $v = v_0$
- $V'(G) = \{v_0\}$
- Call $DFS(v_1)$



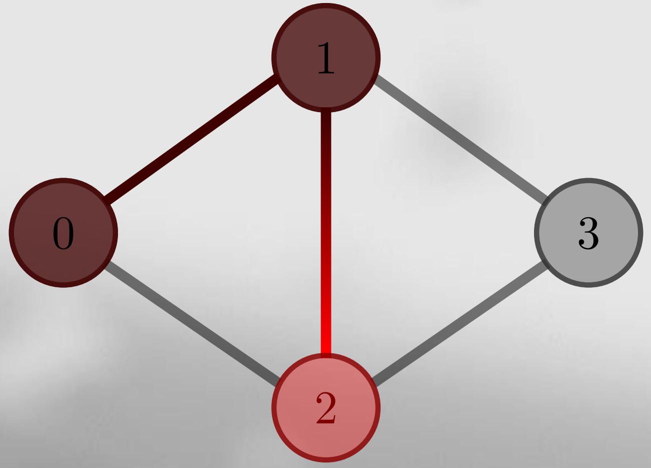
DFS Example - $DFS(v_1)$

- $v = v_1$
- $V'(G) = \{v_0, v_1\}$
- Call $DFS(v_2)$



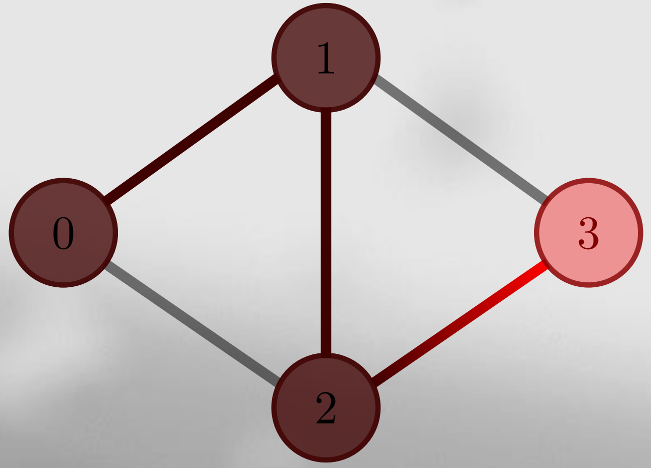
DFS Example - $DFS(v_2)$

- $v = v_2$
- $V'(G) = \{v_0, v_1, v_2\}$
- Call $DFS(v_3)$

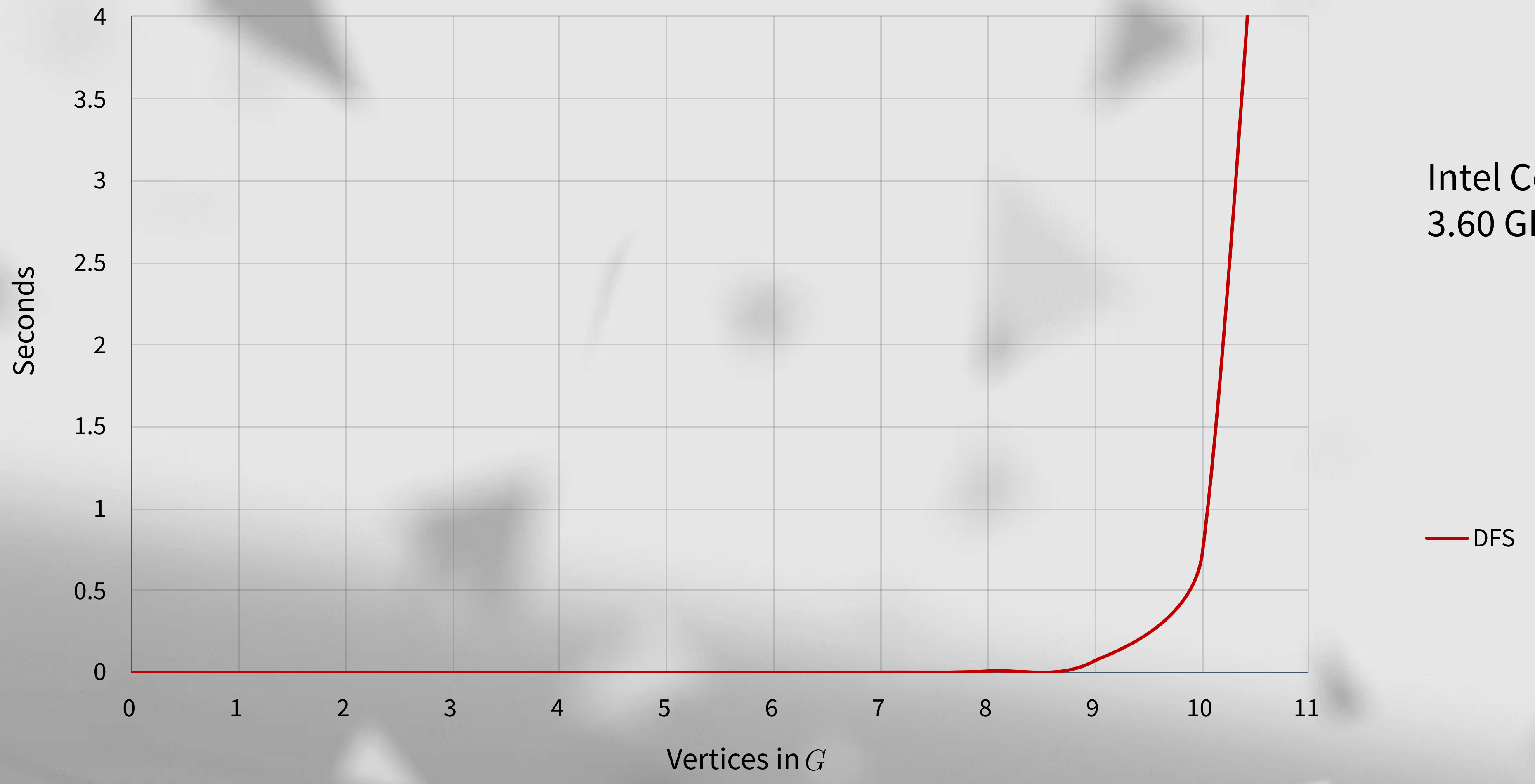


DFS Example - $DFS(v_3)$

- $v = v_3$
- $V'(G) = \{v_0, v_1, v_2, v_3\}$
- The size of $V'(G)$ is 4. Hamiltonian Path found.



DFS – Performance Analysis



Intel Core i7-7700
3.60 GHz

Let’s bash DFS for a sec

- Multiple repeated function calls
- We have to check if we visited a vertex or not
- This is naïve brute-force. We aren’t taking advantage of any “properties”.
- We can do better… *much better*.

Dynamic Programming (DP)

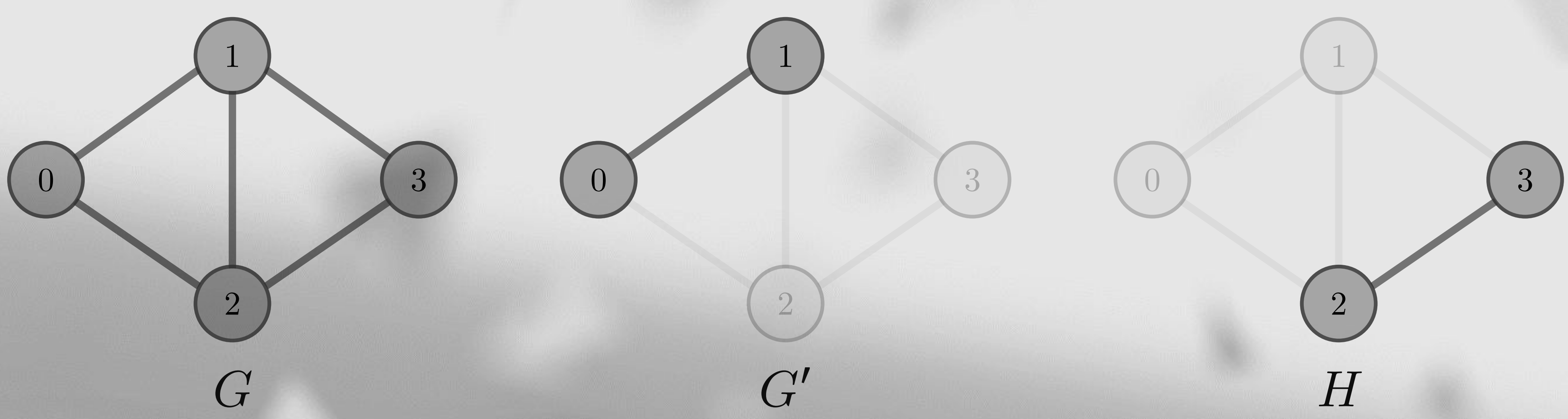
- Mathematical Optimisation by Richard Bellman
- Break a problem down into easier “sub-problems”, solve those, and use the result to solve the original problem.
- “Sub-problems” are broken down into even easier “sub-problems” if possible, recursively.

Held-Karp Algorithm

- Proposed by Michael Held and Richard Karp, as well as independently by Richard Bellman in 1962.
- Utilises DP to solve "sub-problems" of a graph, preventing repeating traversals if a solution is already known.
- Reduces DFS's $O(N!)$ time to $O(2^N \times N^2)$. A significant improvement.
- This was mainly for solving TSP (Travelling Salesman Problem). But the variant here will solve for Hamiltonian Paths.

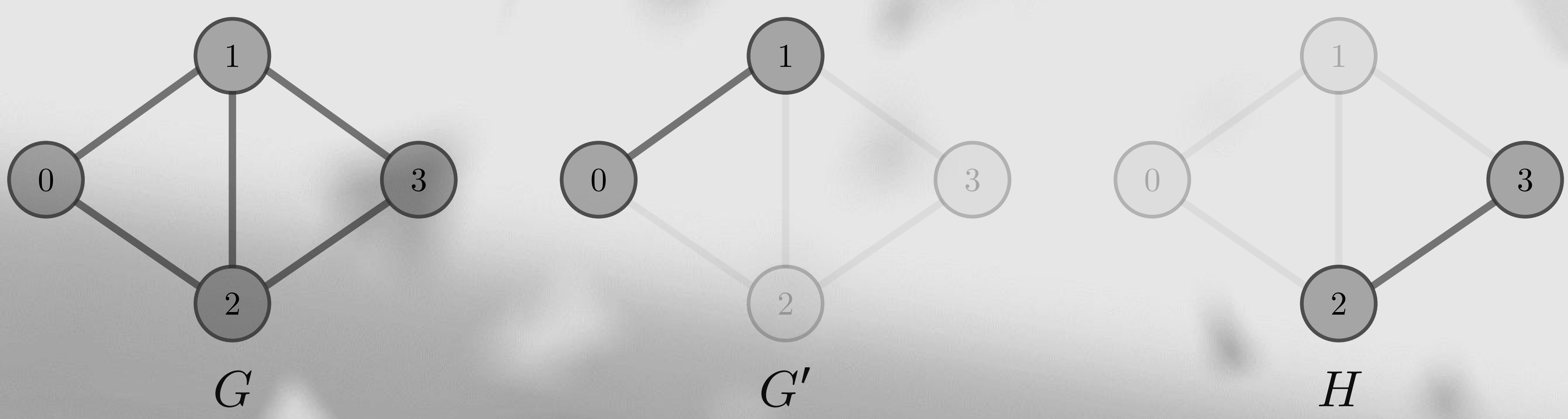
Held-Karp – An Observation

- **Observation:** Assume a graph G, a subgraph G', and $H = G - G'$.
- If there is a Hamiltonian Path in G' and a vertex in G' is adjacent to a vertex v in H in G, then there is a Hamiltonian Path in a subgraph $G' + v$.



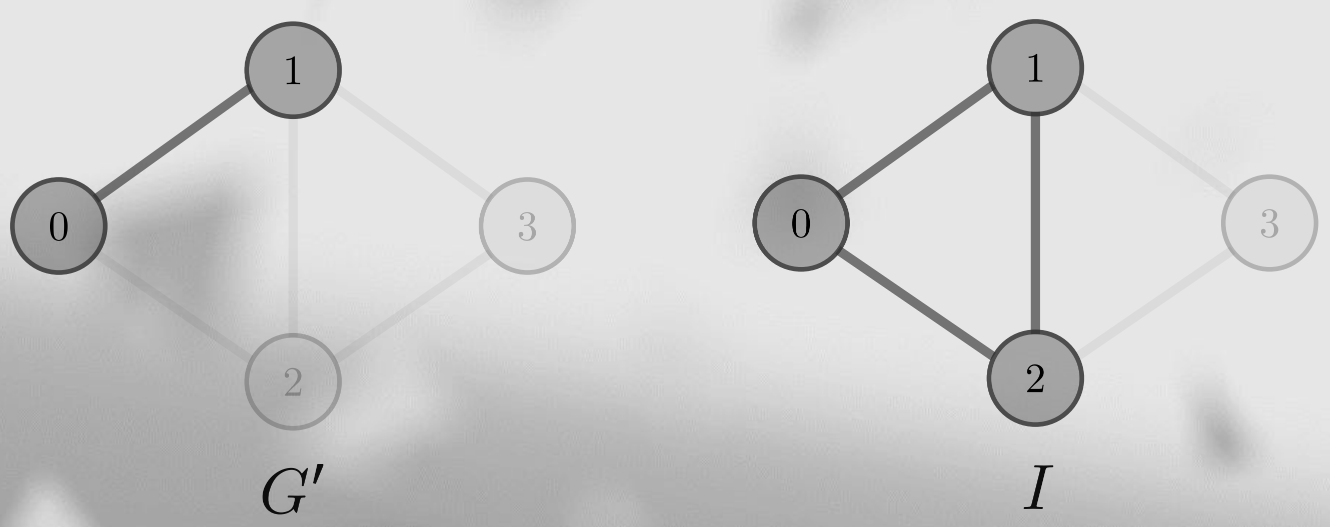
Held-Karp – Example

- Assume a graph G, a subgraph G', and H shown below.
- It's trivial to tell that G' has a Hamiltonian Path $\{v_0, v_1\}$.



Held-Karp – Example

- Now let's look at a new sub-graph, I where $V(I) = \{v_0, v_1, v_2\}$.
- We know there was a Hamiltonian Path in G'. I has the same vertices plus v_2. Since any vertex in G' (v_0 or v_1) can reach v_2, it also has a Hamiltonian Path.

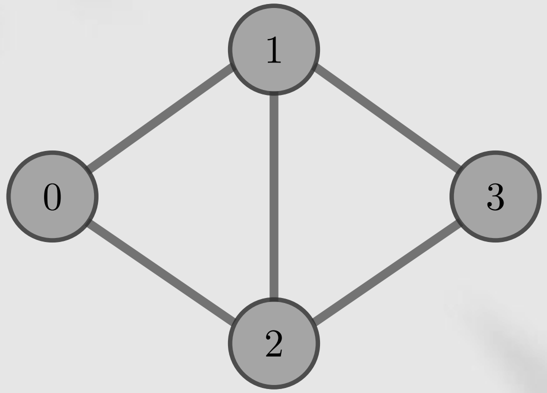


bitDP

- bitDP = **Bit D**ynamic **P**rogramming (ビット(BIT)動的計画法(どうてきけいかくほう))
- Use a DP table where **vertices** go on one side and **bitmasks** go on the other.
    - Bitmask represents subgraphs of G.
- Table is sized $N \times 2^N$.
    - e.g. Graph with 4 vertices has 16 subgraphs, from 0000 to 1111.
- At the final mask (1111), if **any** value is set to 1, there is a Hamiltonian Path in the graph G!

bitDP – Example (Reading the table)

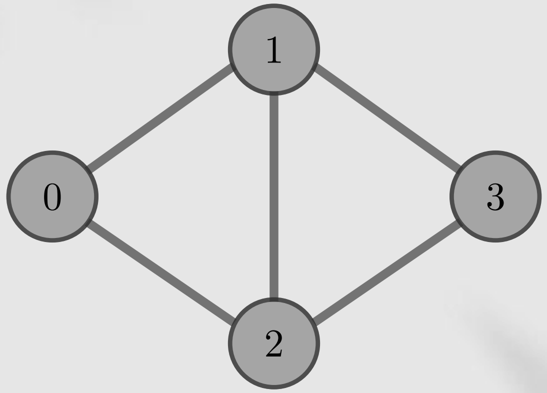
- Make a bitDP table based on the graph:



	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Vertex/Mask	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

bitDP – Example (Reading the table)

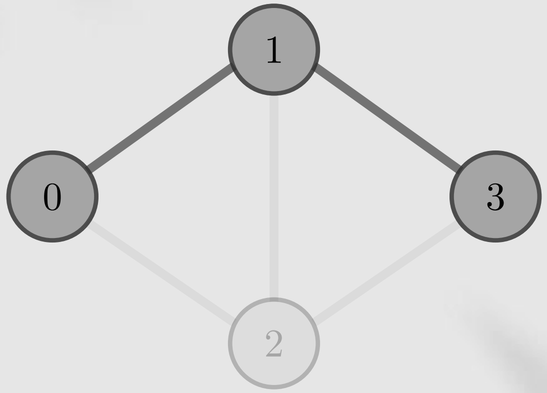
- Make a bitDP table based on the graph:



	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Vertex/Mask	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	0	1	0	1	0	1	0	1	0	0	0	1	0	1	0	1
1	0	0	1	1	0	0	1	1	0	0	1	0	0	0	1	1
2	0	0	0	0	1	1	1	1	0	0	0	0	1	0	1	1
3	0	0	0	0	0	0	0	0	1	0	1	1	1	1	1	1

bitDP – Example (Reading the table)

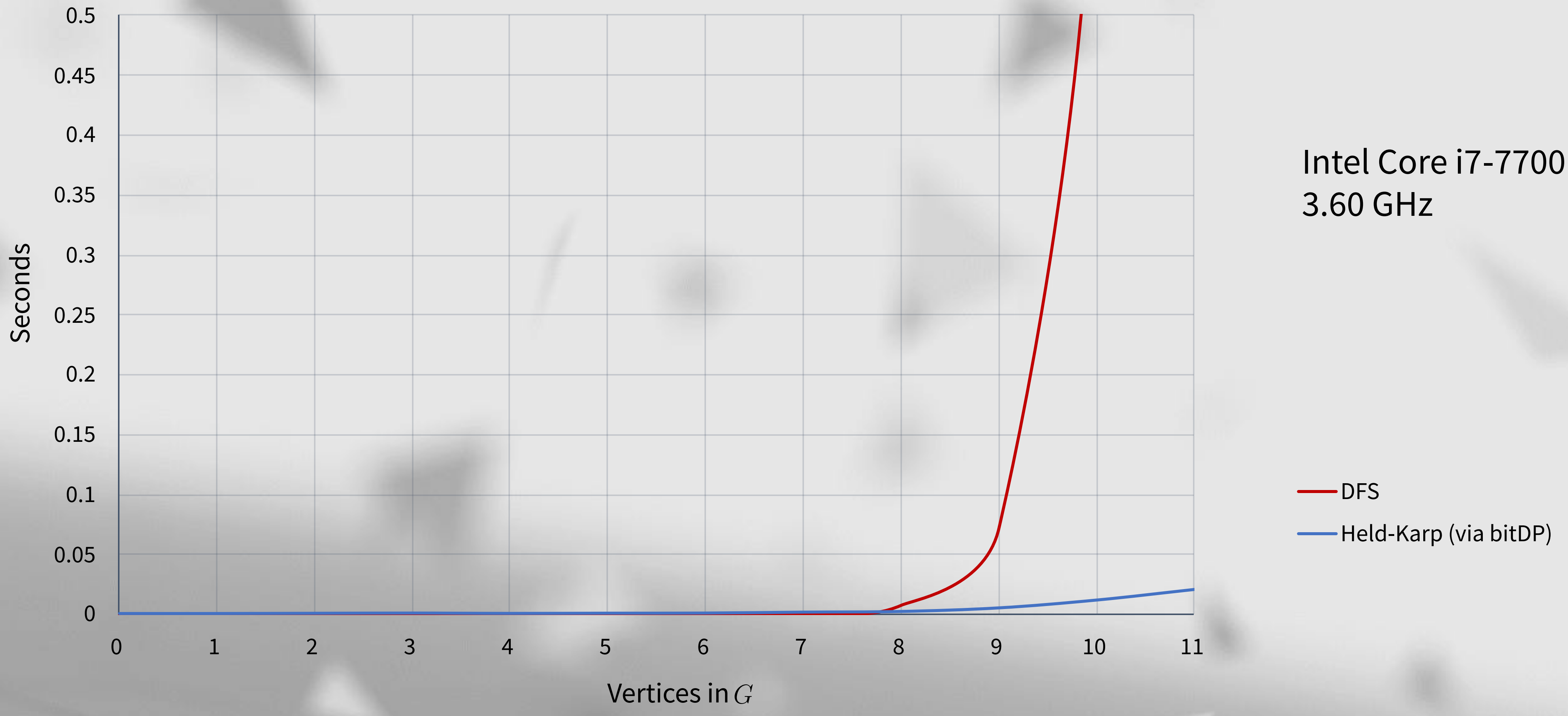
- Consider Mask at 0xB (1011):
  - Vertices Visited: 0, 1, 3



1001	1010	1011	1100	1101
9	A	B	C	D
0	0	1	0	1
0	1	0	0	0
0	0	0	1	0
0	1	1	1	1

- Is there a path between those three that:
  - Ends at 0? Yes
  - Ends at 1? No
  - Ends at 3? Yes

Held-Karp (via bitDP) – Performance Analysis



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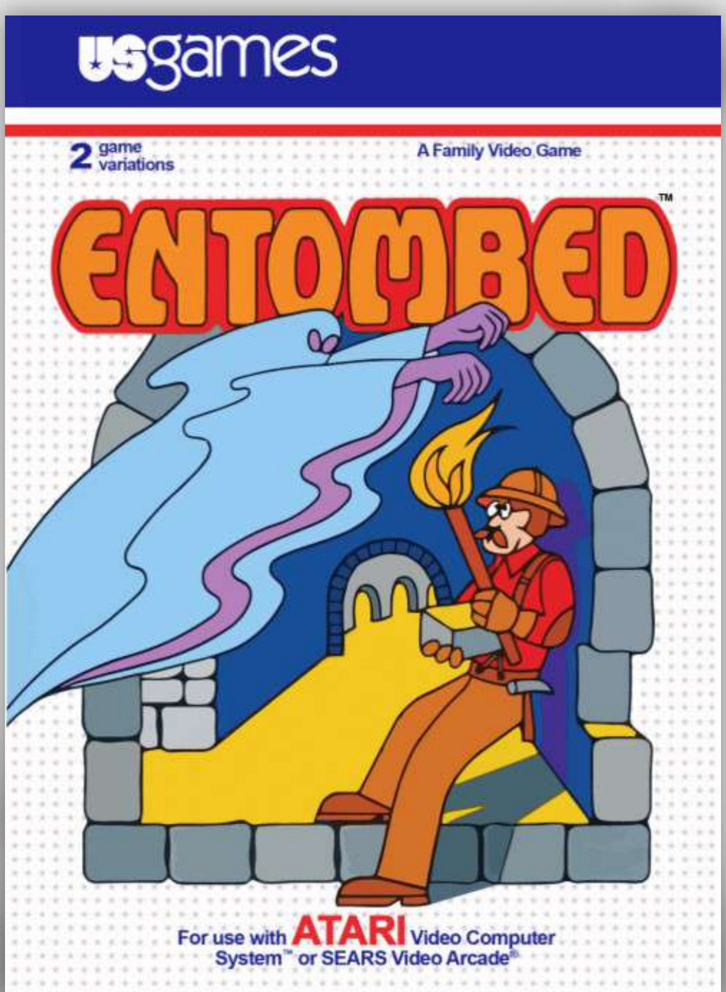
Honourable Mention

Maze Generation, Part II

You thought I was done…

Entombed for Atari 2600 (1982)



Entombed for Atari 2600

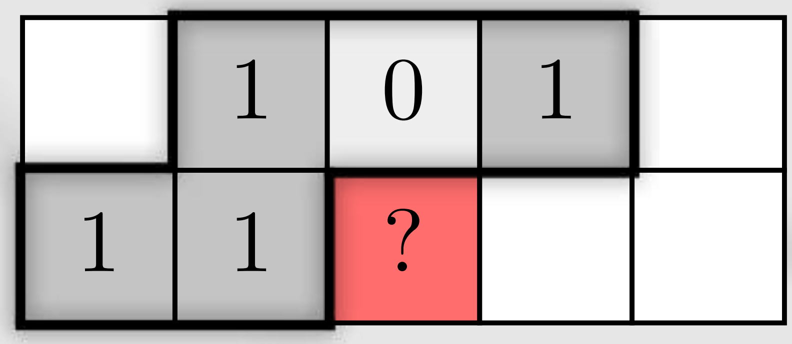
- Released in 1982.
- Simple design. Player moves through a maze trying to avoid enemies. Contact with enemies results in a game over.
- Maze moves upwards.
- If a player is stuck in a dead end, it's also a game over.

Entombed for Atari 2600 – The Technical Details

- Storing all possible mazes in memory is impossible.
- Mazes were generated “on-the-fly”.
- Right side is just a mirrored version of the left side.
- Didn’t use Disjoint-Sets with Union-Find. How did they do it?

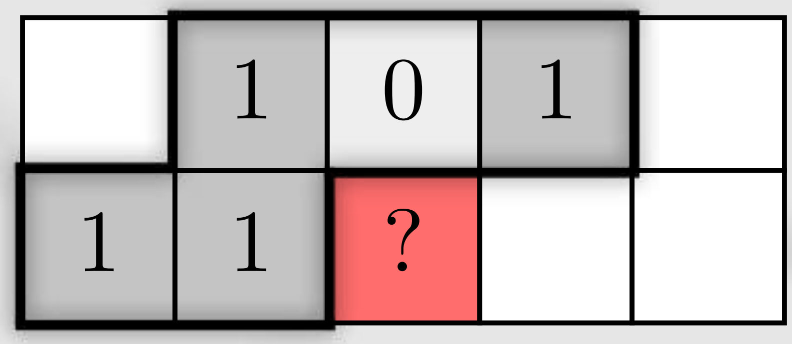
Entombed for Atari 2600 – Maze Generation

- Programmer was **drunk** and developed an "algorithm" for it.
- A cell is set by looking at 5 nearby squares, then looking up information in a lookup table.
- Generates a playable maze… every time… somehow.

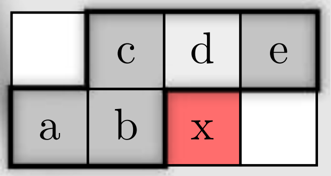


Entombed for Atari 2600 – Maze Generation

- Why does this work? **No one knows why.**
- When programmer was interviewed, he said it came from another programmer.
- Said “He told me it came upon him when he was drunk and whacked out of his brain”.
- It’s even on the Wikipedia page for “List of unsolved problems in computer science”.



Entombed for Atari 2600 – Lookup Table



a	b	c	d	e	x
0	0	0	0	0	1
0	0	0	0	1	1
0	0	0	1	0	1
0	0	0	1	1	?
0	0	1	0	0	0
0	0	1	0	1	0
0	0	1	1	0	?
0	0	1	1	1	?
0	1	0	0	0	1
0	1	0	0	1	1
0	1	0	1	0	1
0	1	0	1	1	1
0	1	1	0	0	?
0	1	1	0	1	0
0	1	1	1	0	0
0	1	1	1	1	0

a	b	c	d	e	x
1	0	0	0	0	1
1	0	0	0	1	1
1	0	0	1	0	1
1	0	0	1	1	?
1	0	1	0	0	0
1	0	1	0	1	0
1	0	1	1	0	0
1	0	1	1	1	0
1	1	0	0	0	?
1	1	0	0	1	0
1	1	0	1	0	1
1	1	0	1	1	?
1	1	1	0	0	?
1	1	1	0	1	0
1	1	1	1	0	0
1	1	1	1	1	0

How does it relate to Graph Theory?

- It’s unsolved, and we know other maze generation algorithms are constructed from graphs, maybe there’s an explanation that involves Graph Theory?
- Apparently, you have to be drunk to make cool stuff…

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Discussion

Questions

- Given a 3D model M of n vertices, how many triangles are drawn if done via Triangle List?
- What is the $lg^*(2^{2^{65536}})$? Alternatively, what is the $lg^*({}^{6}2)$?
- What does bitDP stand for?

Graph Theory Applications in Video Games

Clara Nguyễn

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